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A Short Communication on the Asymptotic Behaviour of the Linear System $\dot{x} = -x$

Ogbonna Nnamuchi

Department of Mathematics

Chukwuemeka Odumegwu Ojukwu University Uli Campus Anambra State Nigeria

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Abstract: **This short communication examines the asymptotic behaviour of the scalar linear differential equation $\dot{x} = -x$. Despite its simplicity, it serves as a canonical model for exponential decay and global asymptotic stability in dynamical systems theory. The analysis includes an explicit solution, phase-line representation, and its role as a local linearization prototype for nonlinear systems.**

Keywords: **asymptotic behavior, canonical model, global asymptotic, dynamical systems theory.**

1. INTRODUCTION

The system

$$\dot{x} = -x \qquad (1)$$

is fundamental in dynamical systems theory and appears in models such as thermal cooling, RC electrical circuits, and local stability analysis of nonlinear systems (Strogatz, 2015). Its importance lies in its role as the simplest representation of exponential convergence toward equilibrium.

2. ANALYTICAL SOLUTION

We solve:

$$\frac{dx}{dt} = -x.$$

Separating variables:

$$\frac{dx}{x} = -\ dt.$$

Integrating:

$$\ln|x| = -t + C.$$

Hence:

$$x(t) = x_0 e^{-t}.$$

3. ASYMPTOTIC BEHAVIOUR

Taking limits:

$$\lim_{t\to\infty} x(t) = 0.$$

Thus, the equilibrium $\dot{x} = 0$ is globally asymptotically stable (Khalil, 2002). Furthermore, the convergence is exponential:

$$|x(t)| \leq |x0|e^{-t}.$$

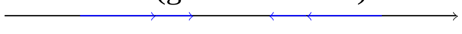
4. PHASE-LINE ANALYSIS

The vector field is $f(x) = -x$.

4.1 If $x > 0$, then $\dot{x} < 0$

4.2 If $x < 0$, then $\dot{x} > 0$

$x = 0$ (global attractor)



5. CONNECTION TO LINEARIZATION THEORY

For nonlinear systems:

$$\dot{x} = f(x), \qquad f(0) = 0,$$

linearization gives:

$$\dot{x} \approx f'(0)x.$$

If $f'(0) < 0$, the local dynamics reduce to $\dot{x} \; - x$

establishing asymptotic stability near equilibrium (Perko, 2001).

6. CONCLUSION

The system $\dot{x} = -x$ represents the fundamental mechanism of asymptotic stability: exponential decay toward equilibrium. Despite its simplicity, it underpins linear stability theory and the local behaviour of nonlinear systems.

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